CALCULATION OF THE "CRITICAL THICKNESS" OF A PLATE IN THE PRESENCE OF LOCAL SURFACE HEATING

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The "critical thicknesses" of cooled plates exposed to local surface heat fluxes are determined.

In [1] attention was drawn to the existence of a "critical thickness," at which in the presence of surface cooling the temperature in the center of a local heat source reaches a minimum. In [2] the effect of certain factors on the "critical thickness" of plates and cylindrical shells was investigated in relation to volume sources.

As one of the authors of [1], V. I. Krylovich, correctly pointed out, in [2] the analytic expressions relate only to volume sources, and therefore the analysis is not directly applicable to the case of an external local heat flux (boundary condition of the second kind). At the same time, in many cases it is desirable to determine the "critical thickness" precisely for a surface local heat flux. The present paper is devoted to the particular solution of this problem.

We will determine the temperature field in a plane wall cooled on one side and heated by a local heat flux on the other. Two typical cases of local heating will be examined, namely, strip (q(x)) and circular (q(r)) sources of arbitrary symmetrical profile (see Fig. 1). In both cases at a great distance from the heating center x = 0 or r = 0 the heat flux intensity tends to zero.

In the first case the stationary temperature field is described by the following system of equations:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0,\ 0\leqslant x<\infty,\ 0\leqslant y\leqslant\delta, \tag{1}$$

$$T|_{x=\infty}=0, \tag{2}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0}=0, \tag{3}$$

$$\lambda\left.\frac{\partial T}{\partial y}\right|_{y=0}=-q(x), \tag{4}$$

$$\lambda\left.\frac{\partial T}{\partial y}\right|_{y=\delta}=-\alpha T. \tag{5}$$

Similarly, in the second case we have

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial y^2}=0,\ 0\leqslant r<\infty,\ 0\leqslant y\leqslant\delta, \tag{6}$$

$$T|_{r=\infty}=0, \tag{7}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0}=0, \tag{8}$$

$$\lambda\left.\frac{\partial T}{\partial y}\right|_{y=0}=-q(r), \tag{9}$$

$$\lambda\left.\frac{\partial T}{dy}\right|_{y=\delta}=-\alpha T. \tag{10}$$

The solution of system (1)–(5), using the Fourier cosine transformation, is easily obtained in the form

$$T(x,\ y)=\sqrt{2/\pi}\times\int_0^\infty\frac{\bar q(u)\left[u\,\mathrm{ch}\,u(\delta-y)+\frac{\alpha}{\lambda}\,\mathrm{sh}\,u(\delta-y)\right]}{\lambda u\left(u\,\mathrm{sh}\,u\delta+\frac{\alpha}{\lambda}\,\mathrm{ch}\,u\delta\right)}\times\cos(xu)\,du. \tag{11}$$

For system (6)–(10), applying a zero-order Hankel integral transformation, we obtain

$$T(r,\ y)=\int_0^\infty\frac{\bar q(\sigma)\left[\sigma\,\mathrm{ch}\,\sigma(\delta-y)+\frac{\alpha}{\lambda}\,\mathrm{sh}\,\sigma(\delta-y)\right]}{\lambda\left(\sigma\,\mathrm{sh}\,\sigma\delta+\frac{\alpha}{\lambda}\,\mathrm{ch}\,\sigma\delta\right)}\times J_0(\sigma r)\,d\sigma. \tag{12}$$

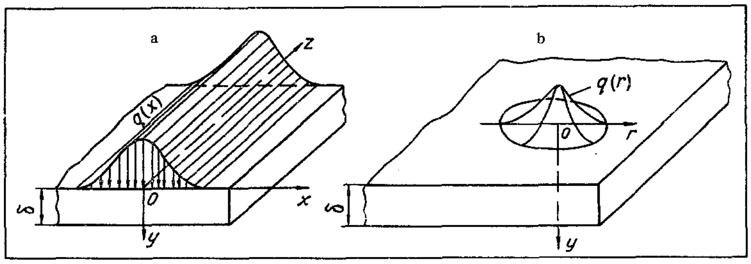

Fig. 1. Diagrams illustrating the action of strip (a) and circular (b) local heat fluxes.

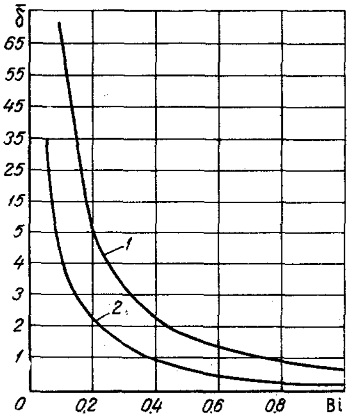

Fig. 2. Relative critical thickness as a function of Bi: 1) for a circular heat flux in accordance with (17); 2) for a strip heat flux in accordance with (16).

The functions $\bar{q}(u)$ and $\bar{q}(\sigma)$ are given by the relations

$$\bar{q}(u) = \sqrt{2/\pi}\int_0^\infty q(x)\cos(xu)\,dx,\quad \bar{q}(\sigma) = \int_0^\infty q(r)\,rJ_0(\sigma r)\,dr. \qquad (13)$$

Obviously, in both cases the maximum temperature will occur on the heating surface y = 0. It follows from (11) and (12) that for given q(x) or q(r) and α/λ the values of the temperature at the surface y = 0 are some function of the thickness δ.

In both cases the extrema of δ can be determined if we differentiate (11) and (12) with respect to δ and equate the results to zero.

Assuming the possibility of differentiation with respect to δ in the integrand of (11) and (12) at y = 0, we obtain the folowing two equations for the values of δ determining the extrema of the temperatures at the heating surface y = 0 for the heat flux distributions considered:

$$\int_0^\infty \frac{(\sigma^2 - \alpha^2/\lambda^2)\,J_0(\sigma r)}{(\sigma\,\mathrm{sh}\,\sigma\delta + \alpha/\lambda\,\mathrm{ch}\,\sigma\delta)^2}\,\bar{q}(\sigma)\,d\sigma = 0. \qquad (14)$$

$$\int_0^\infty \frac{(u^2 - \alpha^2/\lambda^2)\cos ux}{(u\,\mathrm{sh}\,u\,\delta + \alpha/\lambda\,\mathrm{ch}\,u\,\delta)^2}\,\bar{q}(u)\,du = 0, \qquad (15)$$

When specific q(x) and q(r) are given, the determination of the points on the surface with maximum temperatures from (11) and (12) and then, from (14) and (15), the corresponding values of δ, at which the temperatures at these points have a minimum, does not involve any fundamental difficulties.

As an example we will consider the cases

$$q(x) = q_0\ \ 0 \leqslant x \leqslant l,\ \ q(x) = 0\ \ l < x, \qquad (16)$$

$$q(r) = q_0\ \ 0 \leqslant r \leqslant l,\ \ q(r) = 0\ \ l < r. \qquad (17)$$

Substituting (16) and (17) into (13) and then into (14), (15), we obtain the following equations for the relative critical thicknesses $\bar{\delta} = \delta/l$:

$$\int_0^\infty \frac{(\bar{u}^2 - \mathrm{Bi}^2)\sin\bar{u}}{\bar{u}\,(\bar{u}\,\mathrm{sh}\,\bar{u}\,\bar{\delta} + \mathrm{Bi}\,\mathrm{ch}\,\bar{u}\,\bar{\delta})^2}\,d\bar{u} = 0, \qquad (18)$$

$$\int_0^\infty \frac{(\bar{\sigma}^2 - \mathrm{Bi}^2)\,J_1(\bar{\sigma})}{(\bar{\sigma}\,\mathrm{sh}\,\bar{\sigma}\,\bar{\delta} + \mathrm{Bi}\,\mathrm{ch}\,\bar{\sigma}\,\bar{\delta})^2}\,d\bar{\sigma} = 0. \qquad (19)$$

The real positive values of the roots $\bar{\delta}$ of Eqs. (18) and (19), depending on the given values of Bi, are found numerically by the chord method. The integrals were evaluated on a computer using Simpson's rule with automatic selection of the integration step. The results are presented in Fig. 2 in the form of curves showing the relative critical thickness as a function of the number Bi.

It would be desirable to make similar investigations for cylindrical and spherical shells, for the case of a moving temperature field and for other laws of variation of the surface source profile. These will be the subject of future communications.

NOTATION

T is the difference $T_i - T_c$, where T_i is the temperature at a given point, T_c the temperature of the medium; α is the heat transfer coefficient; q(u) is the cosine transform of the function q(x); $\bar{q}(\sigma)$ is the zero-order Hankel transform of the function q(r); J_n is the Bessel function of the first kind of real argument of order n; $\bar{\delta}$ is the relative thickness, $\bar{\delta} = \delta/l$; Bi = $\alpha l/\lambda$.

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